Global self-excited oscillations in a two-dimensional heated jet : a numerical simulation

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The aim of this work was to develop a numerical methodology to gain insight in the low-density jet behaviour with a *nonlinear* approach. Numerical simulations are shown to differentiate convective and absolute instability regimes and to capture a self-excited global mode in an open flow : the 2D hot jet.

The first part is devoted to numerical methodology and its validation on this unsteady problem which is known to be noise sensitive. The second part presents numerical results. They confirm theoretical and experimental results on the development of self-excited global oscillations of the jet column when the density ratio is lower than its critical value. The global mode and its associated Hopf bifurcation are identified.

1. Numerical methodology

The time-dependent compressible Euler equations, with boundary conditions, are time marched with a 5-stage 2nd-order Runge-Kutta scheme (Jameson & al. 1981) in conservative variables with a finite volume method. The spatial discretization is based on 2nd-order central differences. The used calculation domain is 18-D long (with D the exit jet height) and 10-D wide. Mesh refinement is used in the potential core region. For precision, a typical large scale vortical structure in the finely resolved part of the fine grid (286x213) is approximately described with 40x40 gridpoints. The boundary conditions have been chosen in the following way : slip conditions at the lateral boundaries, a non-reflecting condition at the outlet and a reservoir condition at the inlet with prescribed velocity and density profiles.

The independence of the global mode frequency has been controlled towards grid resolution, outlet boundary condition location and simulation time. Two calculations were made with a coarse mesh (143x107) and a domain length of 18-D and 32-D. Frequencies were identical within the frequency resolution of 3%. Then, simulations were performed for a long enough time integration to check the statistically stationnarity of the frequency within 1% (the resolution of the Fourier analysis).

2. Numerical experiments

Our laminar jet configuration has been defined with Yu & Monkewitz (1990) paper. For numerical reasons, a small convection velocity outside the jet was adopted yielding a velocity ratio of $\Lambda = (U_j - U_\infty)/(U_j + U_\infty) = 0.93$ in all calculations. In that case, the critical density ratio predicted by the linear theory with zero Mach number is around $S_c = \rho_{jc}/\rho_\infty = 0.6$. According to Monkewitz & Sohn (1988), S_c decreases with Mach number in axisymmetric inhomogeneous jets. Hence, a density ratio of $S = \rho_j/\rho_\infty = 0.4$ was expected to give an absolute instability regime for our 0.15 jet Mach number.

2.1 Absolute and convective instability

The hot (S=0.4) and cold (S=0.9) jet near-field velocity spectrum are shown in figure 1 (left). The same behaviour found in experimental work (Yu & Monkewitz 1993) is exhibited. A line dominated spectrum with harmonic contents is detected, characteristic of absolute instability, whereas for convective one, broadband spectrum with small peaks is observed. After a transient, a quasi-periodic longitudinal velocity on the jet centerline is seen for S=0.4, see figure 1 (right). The same qualitative structure is noticed everywhere in the potential core.

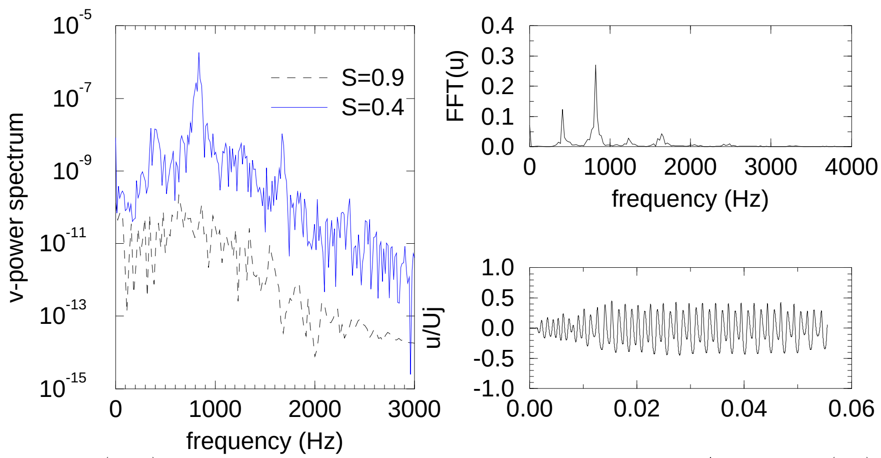

FIGURE 1. (left) Transversal velocity power spectrum at x/D=0.6 : (----) S=0.9, (—) S=0.4, (right) Longitudinal velocity signal and spectrum at x/D=3.5, S=0.4

Some comparisons were made with the experimental work of Yu & Monkewitz (1993). In figure 2, the weak dependence of the Strouhal number with jet velocity U_j is verified. So, the main frequency is well correlated to the jet preferred mode. The value of the Strouhal number, St=f_p.D/U_m=0.30 is closed to the Yu and Monkewitz's (1993) value St=0.32. Moreover, the main frequency is also nearly independent of the density ratio S when $S < S_c$, see figure 2. This is a basic characteristic of a normal Hopf bifurcation (Bergé *et al.* 1984).

2.2 The global mode of the jet

In this section, the jet asymptotic states for supercritical density ratio are investigated. For sufficiently low density ratio, numerical simulations have shown that, oscillations seem to saturate to a global self-excited mode (figure 1). Following Huerre and Monkewitz (1990), two classes of experiments are suitable for the identification of

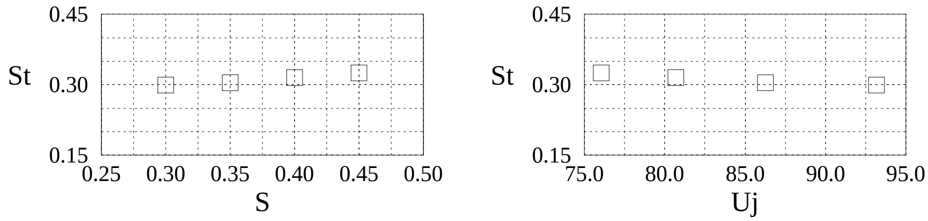

FIGURE 2. Strouhal number versus : (left) density ratio S, (right) centerline jet velocity U_j.

these global modes. The first class, only supports evidence for the existence of self-excitation. The second one yields conclusive proof of self-excitation. Presently, first class experiments have been used :

1. The examination of single-point spectrum in the jet considered in figure 1 suggests the presence of a limit-cycle.

2. The response of convective instability is proportional to the forcing amplitude whereas, the response of absolute instability is intrinsic *i.e.* independent of the forcing amplitude for low level forcing (Sreenivasan & al. 1989). In figure 3, the response amplitude of the jet to white-noise upstream excitation versus the amplitude excitation is plotted for two density ratio. For S=0.9, the response is typical of convectively unstable flows. For S=0.4, the response is quasi independent of the amplitude excitation suggesting a self-excited system. Response frequency f_r to periodic forcing at f_f, figure 3 (right), leads to the same conclusion.

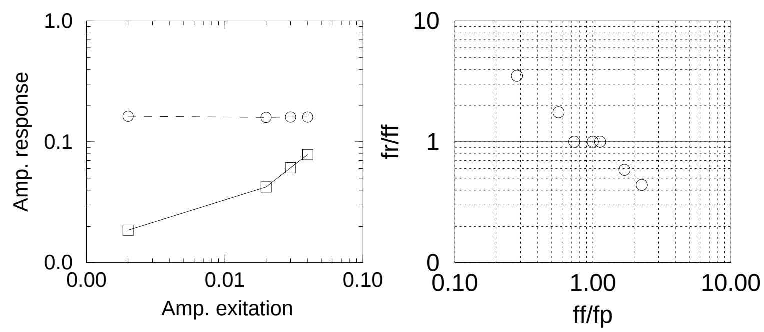

FIGURE 3. Jet response to excitation : (left) White noise excitation with (—) S=0.9 and (– –) S=0.4, (right) Periodic forcing at f_f for S=0.4.

3. Calculations were made with different density ratio. Figure 4 shows a typical example of the jet global mode saturation amplitude versus density ratio S and identify a Hopf bifurcation. With a Mach number of 0.15, a critical density ratio $S_c = 0.47$ was found. This is consistent with a calculation for S=0.5 where self-excited oscillations were not observed.

In a subsequent part, second class experiments are planned to be investigated in order to positively caracterize the Hopf bifurcation, by determining the coefficients of the Landau-Stuart model (Provansal *et al.* 1987).

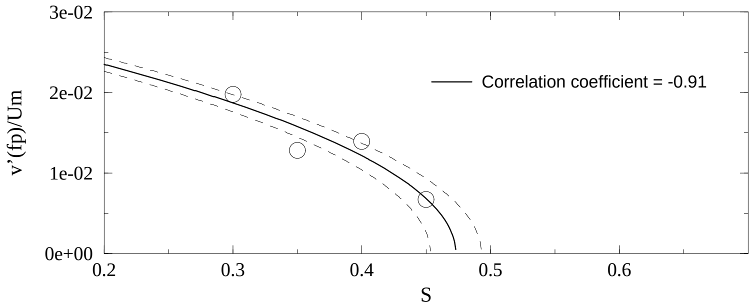

FIGURE 4. Transverse velocity amplitude of the spectral peak f_p versus density ratio S.

3. Conclusions

This first step toward the numerical simulation of a heated two-dimensional jet has been shown to develop a self-excited global mode when an appropriate density ratio is used.

To our knowledge, it is the first time that a numerical simulation of a heated two-dimensional jet identifies a self-excited global mode as a normal Hopf bifurcation and evaluates the critical density ratio for one doublet (Λ, M).

After these results, it should be possible to predict the absolute/convective boundary in the S-M plane with numerical nonlinear computations, and perhaps, to put some light on the large discrepancy found in the low Mach number region of the S-M plane between experimental and theoretical results on round jets, see Sreenivasan *et al.* (1989).

References

BERGÉ, P., POMEAU, Y. & VIDAL, CH. (1984) *L'ordre dans le chaos - Vers une approche déterministe de la turbulence.* Hermann, 1984.

HUERRE, P. & MONKEWITZ, P.A. (1990) Local and global instabilities in spatially developing flows, *Ann. Rev. of Fluid Mech., vol 22, 473-537.*

JAMESON, A., SCHMIDT, W. & TURKEL, E. (1981) , *A.I.A.A. paper 81-1259.*

MONKEWITZ, P.A. & SOHN, K.D. (1988) Absolute instability in hot jets, *A.I.A.A. J. 26, 911-916.*

PROVANSAL, M., MATHIS, C. & BOYER, L. (1987) Bénard-von Karman instability : Transient and forced regimes, *J. Fluid Mech., vol. 182, 1-22, 1987.*

SREENIVASAN, K.R., RAGHU, S. & KYLE, D. (1989) Absolute instability in a variable density round jets, *Exps. Fluids 7, 309-317.*

YU, M.H. & MONKEWITZ, P.A. (1990) The effect of nonuniform density on the absolute instability of two-dimensional inertial jets and wakes, *Phys. Fluids A 2(7), July 1990, 1175-1181.*

YU, M.H. & MONKEWITZ, P.A. (1993) Oscillations in the near field of a heated two-dimensional jet, *J. Fluid Mech. vol. 255, 323-347.*